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HEAT TRANSFER IN SIMPLE MONATOMIC GASES AND IN BINARY MIXTURES OF MONATOMIC GASES

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Abstract—The conductive heat transfer in simple monatomic gases and in binary mixtures thereof has been investigated both experimentally and theoretically for temperature differences up to 300 K and for a Knudsen number range of $10^{-4} < Kn < 10^{1}$. The experiments have been performed in a heat-transfer cell with horizontal parallel flat plates and the theoretical results have been obtained with four moment and eight moment solutions of the Boltzmann equation.

NOMENCLATURE

D, diffusion coefficient;
Kn, Knudsen number;
T_I, temperature of upper (hot) plate;
T_{II}, temperature of lower (cold) plate;
ΔT, $= T_I - T_{II}$, temperature difference between hot and cold plate;
T_0, reference temperature;
$\bar{T}$, intermediate temperature;
Q, heat per unit time;
d, distance of plates;
s, exponent of temperature law for heat conductivity;
f, molecular distribution function;
n, number density;
m, molecular mass;
k, Boltzmann's constant;
q, heat flux;
p, pressure.

Greek symbols
α, accommodation coefficient;
η, viscosity;
λ, heat conductivity;
ξ, molecular velocity.

Subscripts
I, 1, hot plate;
II, 2, cold plate;
i, component of binary mixture;
FM, free molecule;
K, continuum;
L, linear theory.

1. INTRODUCTION

THE CONDUCTIVE heat transfer in rarefied gases has been investigated experimentally and theoretically with success for small temperature differences in previous papers [1–5]. It is the purpose of the present paper to present new theoretical and experimental results for simple monatomic gases and for binary mixtures of monatomic gases in an extended temperature range. Temperature differences up to 300 K in a maximum Knudsen number range of $10^{-4} < Kn < 10^{1}$ have been applied. The experiments have been performed in a heat-transfer cell with horizontal flat plates in order to avoid free convection currents. For the gas enclosed by the two plates a stable situation is obtained, as long as the temperature of the upper plate is higher than the temperature of the lower one. The experimental results are compared with solutions of the Boltzmann equation obtained by a four moment method in the case of simple gases [2, 3] and with an eight moment method in the case of binary gas mixtures [4–6].

2. EXPERIMENTS

A schematic view of the heat-transfer cell with plane plates is shown in Fig. 1. The upper plate is heated electrically. This is achieved with stainless steel sheathed heating wire which after embedding in a spiral groove is vacuum brazed to the outer side of the hot plate. The heat sink is provided by pumping methanol from a constant temperature bath through a spiral groove in the lower plate. The outer diameter of both plates is 275 mm. In order to eliminate disturbances from the edges only the heat conducted from the central plate of the hot plate with a diameter of 100 mm was evaluated. Heat transfer to the surrounding walls of the housing has been minimized by an auxilliary third plate which was located above the hot plate. The temperature difference between hot plate and auxilliary plate was less than 1 K in all experiments. Heat losses to the auxilliary plate have been further reduced by lowering the pressure in the space between these two plates to about 10^{-3} torr. Seven thermocouples located in the central part of the hot plate, three thermocouples in the outer ring and two thermocouples in the auxilliary plate were available to measure the tem-

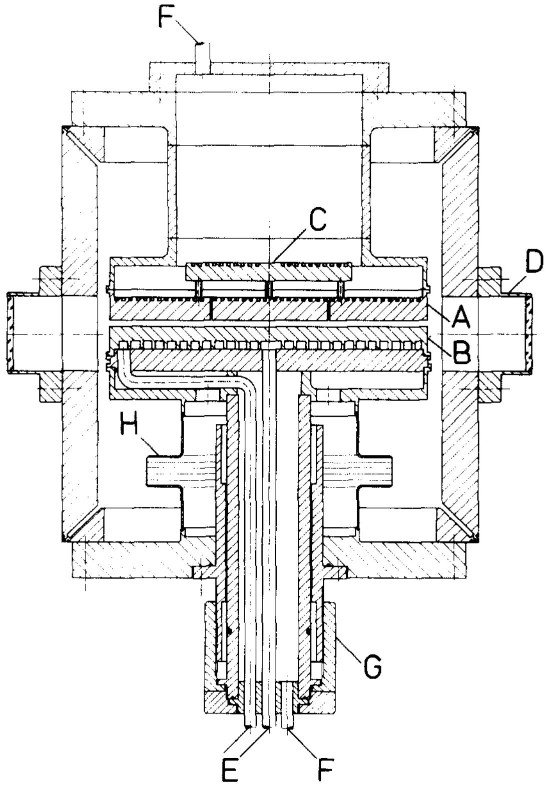

FIG. 1. Schematic view of heat transfer cell with plane plates: A, hot plate; B, cold plate; C, auxilliary plate; D, viewing port; E, cooling fluid; F, to forepump; G, adjustment of plate distance; H, bellows.

perature distribution in the plate surfaces. Electronic controllers were used to minimize all temperature non-uniformities on the plates. At a temperature difference of $T_I - T_{II} = 100\,K$ between hot and cold plate the maximum temperature variation was 0.5 K on the central part of the hot plate and not larger than 1 K on the entire hot plate. The entire system was made of stainless steel. It could be baked at 400°C. The vacuum system consisted mainly of a Varian ion pump with a pumping capacity of 800 l/s. Four sorption pumps and alternatively a two-stage roots-pump are used as forepumps. After a baking period of 48 h at 350°C an ultimate vacuum of 10^{-9} torr was measured on a Varian dual range ionization gauge. Pressure measurements during the heat-transfer experiments were performed with three bakable capacity manometer heads in combination with a Consolidated Vacuum Corporation McLeod manometer and a mercury U-tube manometer. The heat transfer is obtained from the electrical power input. Radiation and other losses are determined at a pressure of approximately 10^{-8} torr. Mass spectrograms could be made with a Balzers quadrupole mass spectrometer which could also be used together with a free molecule probe to determine the species distribution as a function of distance between the plates.

3. RESULTS

In order to check the performance of the heat-transfer cell all measurements in the continuum regime for the

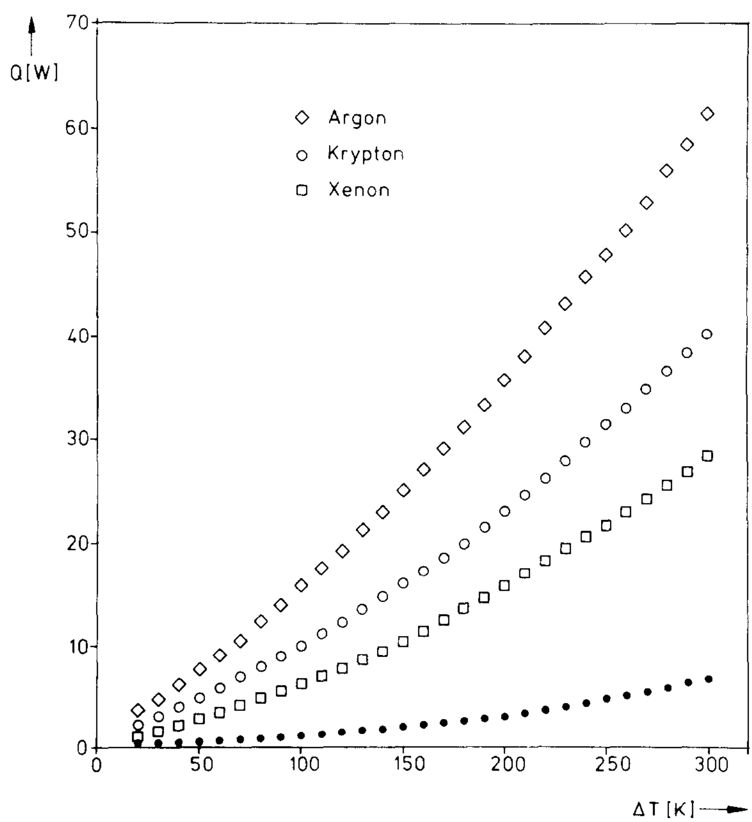

FIG. 2. Heat transfer measured between parallel flat plates at a distance of $d = 0.1$ cm as a function of temperature differences. Losses (.....) were determined at a pressure of 10^{-8} torr.

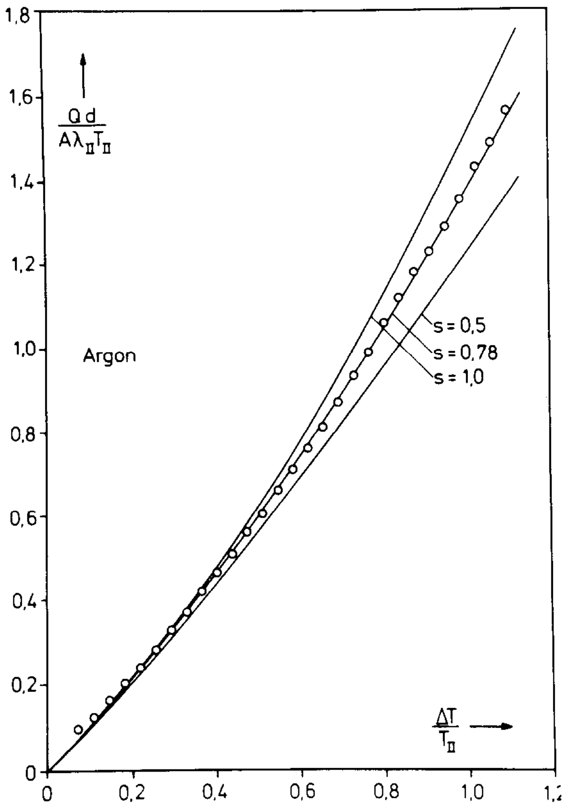

FIG. 3. Dimensionless heat transfer for argon between parallel flat plates as a function of temperature difference.

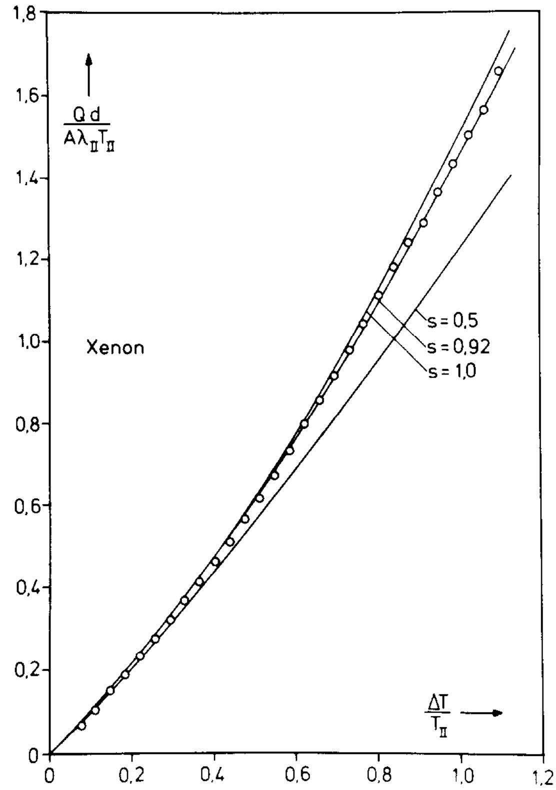

FIG. 5. Dimensionless heat transfer for xenon between parallel flat plates as a function of temperature difference.

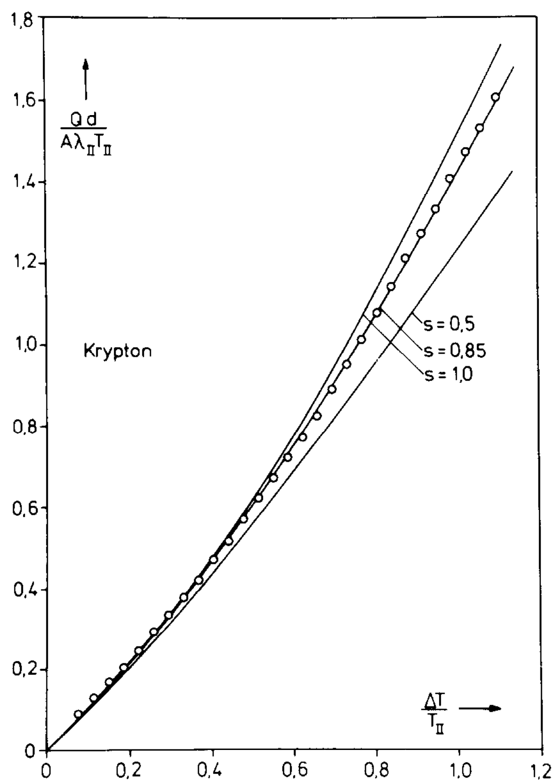

FIG. 4. Dimensionless heat transfer for krypton between parallel flat plates as a function of temperature difference.

monatomic gases Ar, Kr, Xe have been made for a maximum temperature difference $T_I - T_{II} = 300\,K$. The results are shown in Fig. 2. In Figs. 3–5 the measured heat transfer is compared with theoretical results obtained from Fourier's law assuming that the heat conductivity follows the simple power law $\lambda/\lambda_0 = (T/T_0)^s$. The heat transferred Q has been nondimensionalized with the surface A, the plate distance d and the heat conductivity at the cold plate $\lambda_{II} = \lambda(T_{II})$. In Table 1 the values for the quantity s which gave the best fit between experiment and theory are shown in comparison with results of other authors.

Next the heat transfer is investigated as a function of the gas density. The continuum regime can be described with Fourier's law and the free molecule regime with the well known Knudsen formula. In order to obtain a theoretical representation of the heat

Table 1. Experimental values for exponent s

	Ar	Ref.	Kr	Ref.	Xe	Ref.
	0.651	[9]	0.668	[13]	0.71	[9]
	0.766	[7]	0.85	[7]	0.88	[10]
	0.77	[10]	0.85	*	0.92	*
	0.78	*	0.86	[10]	0.92	[7]
s	0.811	[11]				
	0.815	[12]				
	0.823	[12]				
	0.86	[8]				

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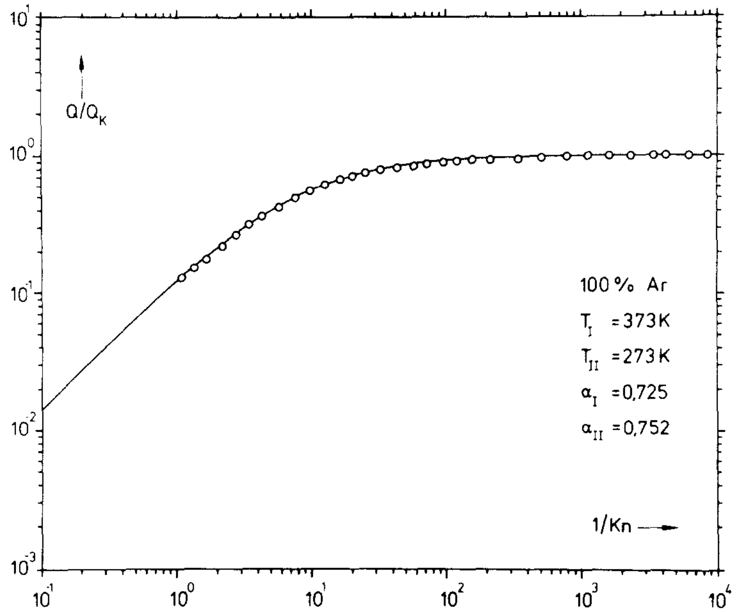

FIG. 6. Heat transfer in argon as a function of the inverse Knudsen number for a temperature difference of 100 K in comparison with the four moment theory [6]: $Q_K = 14.9866$ W, $\eta^{Ar} = 2.092 \cdot 10^{-4}$ g/cm s.

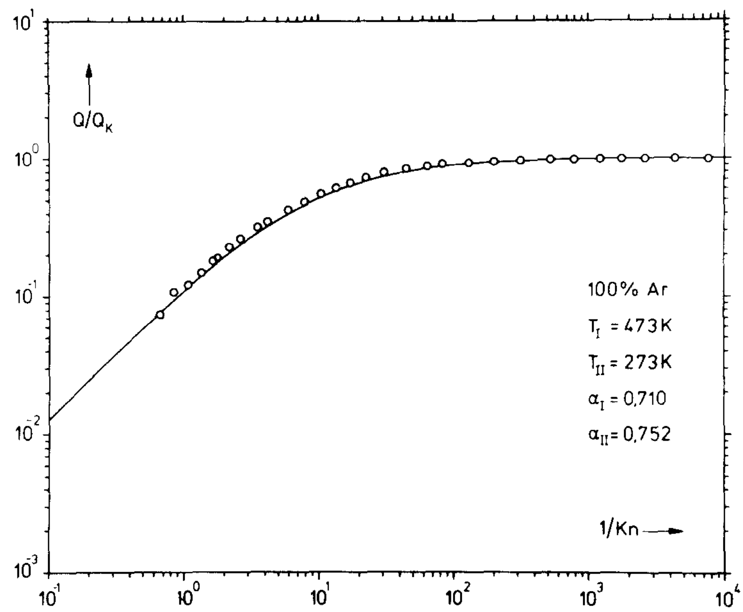

FIG. 7. Heat transfer in argon as a function of the inverse Knudsen number for a temperature difference of 200 K in comparison with the four moment theory [6]: $Q_K = 33{,}5484$ W.

transfer for the entire Knudsen number range a solution is derived from Maxwell's transport equations using an unsymmetrical distribution function [3–5]:

$$f_1^i = n_1^i(m^i/2\pi k T_1^i)^{3/2} \exp(-m^i\xi^{i2}/2kT_1^i)$$
$$f_2^i = n_2^i(m^i/2\pi k T_2^i)^{3/2} \exp(-m^i\xi^{i2}/2kT_2^i).$$

The distribution function f_1^i represents the molecules coming from the hot wall and the distribution function f_2^i represents the molecules coming from the cold wall. The index "i" specifies the distribution functions of the components in a binary mixture. Details of the theoretical calculations are given in [2–6]. It may be mentioned that the accommodation coefficients which enter the theory may be obtained from the Knudsen formula

$$q_{FM} = \alpha(2k/\pi m\bar{T})^{1/2}(T_I - T_{II})p,$$

where $\bar{T}$ is the intermediate temperature of the gas and is given by the expression $\alpha = \alpha_I\alpha_{II}/(\alpha_I + \alpha_{II} - \alpha_I\alpha_{II})$ [7, 8]. In Figs. 6–10 it is shown that a good agreement between experiment and theory is obtained for all

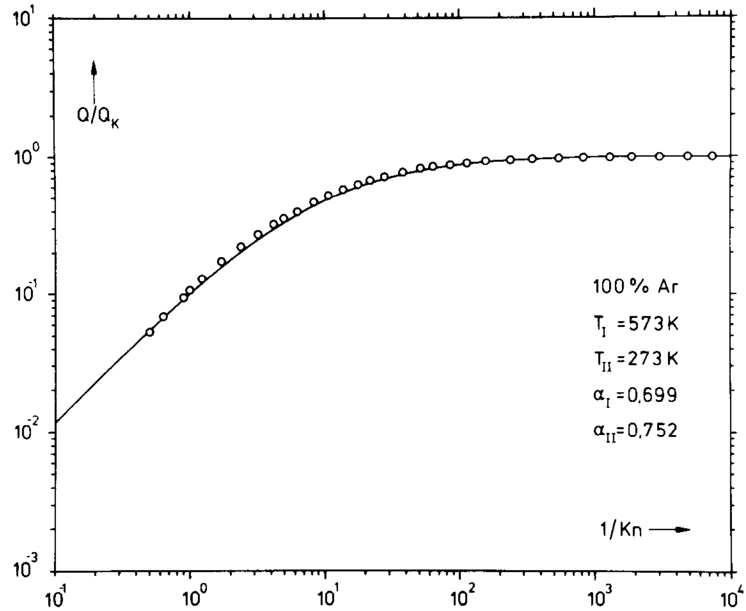

FIG. 8. Heat transfer in argon as a function of the inverse Knudsen number for a temperature difference of 300 K in comparison with the four moment theory [6]: $Q_K = 57.2505$ W.

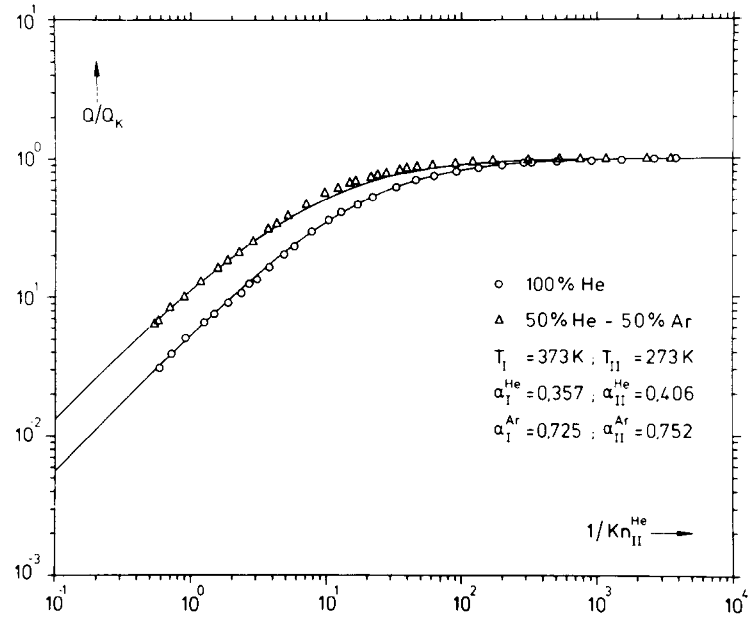

FIG. 9. Heat transfer in helium and in a binary mixture of helium and argon as a function of the inverse Knudsen number for a temperature difference of 100 K in comparison with the eight moment theory [6]: $Q_K^{He} = 108.7735$ W, $Q_K^{50/50} = 40.5990$ W, $\eta_{II}^{He} = 1.822 \cdot 10^{-4}$ g/cm s, $D_{II}^{He\,Ar} = 0.640$ cm^2/s.

Knudsen numbers and for all temperature differences. The dimensionless form of the heat flux Q/Q_K has been obtained with the continuum value Q_K.

The heat transfer in simple monatomic gases with small temperature differences can be given by the simple analytical expression

$$\frac{Q_L}{Q_K} = \frac{1}{1+\frac{15}{4}Kn}.$$

In Fig. 11 a comparison between this result of the linearized moment equations and numerical equations is given.

4. CONCLUSIONS

The experimental results of the present paper show that the heat transfer in simple monatomic gases and in binary mixtures of monatomic gases can be predicted with the moment method in a wide range of temperature differences and Knudsen numbers which include the free molecule regime, the transition regime and the continuum regime. It may be mentioned that the theory used for the binary mixtures predicts a slight separation of both components [4–6]. Concentration measurements with a free molecule probe have given good

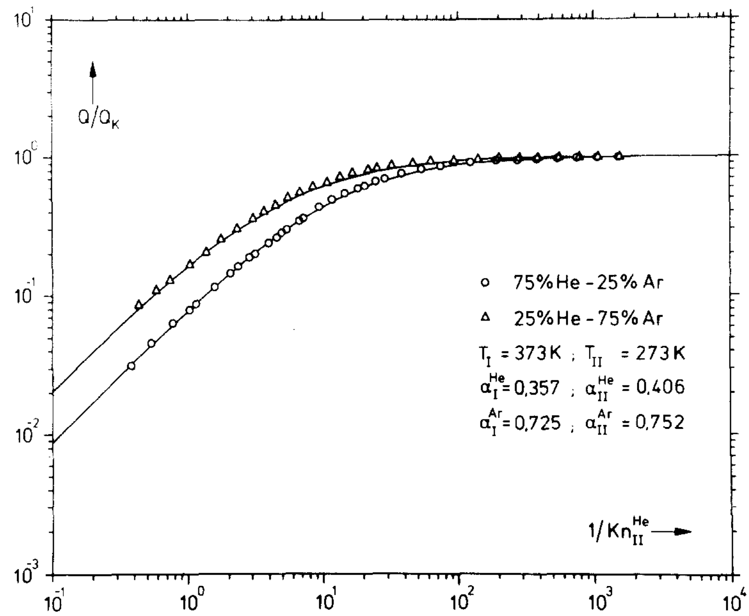

FIG. 10. Heat transfer in binary helium—argon mixtures as a function of the inverse Knudsen number for a temperature difference of 100 K in comparison with the eight moment theory [6]: $Q_K^{75/25} = 65.5329$ W, $Q_K^{25/75} = 25.7304$ W.

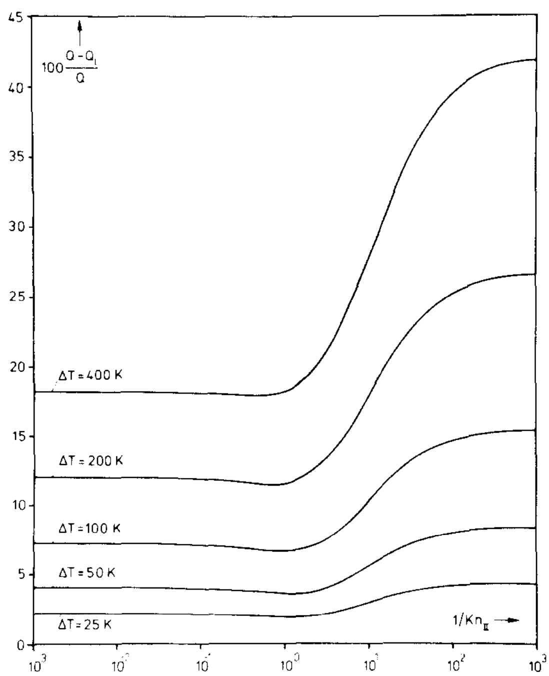

FIG. 11. Error of linearized theory for heat transfer as a function of the inverse Knudsen number with the temperature difference as a parameter.

agreement between experiment and theory for pressures below 0.1 torr. This effect will be discussed in a separate paper.

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REFERENCES

1. A Dybbs and G. S. Springer, Heat conduction experiments in rarefied gases between concentric cylinders, *Physics Fluids* **8**, 1946 (1965).
2. C. Y. Liu and L. Lees, Kinetic theory description of plane compressible Couette flow, in *Rarefied Gas Dynamics*, edited by L. Talbot, Vol. 1, p. 391 (1961).
3. L. Lees and C. Y. Liu, Kinetic theory description of heat transfer from a fine wire, *Physics Fluids* **5**, 1137 (1962).
4. B. T. Yeh and A. Frohn, Heat conduction between parallel flat plates in binary gas mixtures, *Physics Fluids* **16**, 330 (1973).
5. B. T. Yeh and A. Frohn, Heat conduction in binary gas mixtures between concentric cylinders, *Physics Fluids* **16**, 801 (1973).
6. D. Braun, Wärmetransport in Gasen bei großen Temperaturdifferenzen und beliebigen Knudsenzahlen, Dissertation RWTH Aachen, forthcoming.
7. E. H. Kennard, *Kinetic Theory of Gases*. McGraw-Hill, New York (1938).
8. S. Dushman, *Scientific Foundations of Vacuum Technique*. John Wiley, New York (1962).
9. G. S. Springer and E. W. Wingeier, Thermal conductivity of neon, argon and xenon at high temperatures, *J. Chem. Phys.* **59**, 2747 (1973).
10. H. von Ubisch, The thermal conductivities of mixtures of rare gases at 29°C and at 520°C, *Ark. Fys.* **16**, 93 (1959).
11. S. Chapman and T. G. Cowling, *The Mathematical Theory of Non-uniform Gases*. Cambridge University Press, Cambridge (1970).
12. J. H. Jeans, *The Dynamical Theory of Gases*. Dover, New York (1954).
13. F. M. Faubert and G. S. Springer, Measurement of the thermal conductivity of argon, krypton and nitrogen in the range 800–2000 K, *J. Chem. Phys.* **57**, 2333 (1972).

TRANSFERT DE CHALEUR DANS LES GAZ SIMPLES MONOATOMIQUES ET DANS LES MELANGES BINAIRES DE GAZ MONOATOMIQUES

Résumé—Le transfert de chaleur conductif dans les gaz simples monoatomiques et dans leurs mélanges binaires a été étudié à la fois par voie expérimentale et par voie théorique pour des différences de température atteignant 300 K et pour un domaine de nombres de Knudsen vérifiant $10^{-4} < K_n < 10^3$. Les expériences ont été réalisées dans une cellule de transfert thermique limitée par des plaques planes horizontales parallèles et les résultats théoriques ont été obtenus à l'aide des solutions de l'équation de Boltzmann à quatre moments et à huits moments.

WÄRMEÜBERGANG IN EINFACHEN EINATOMIGEN GASEN UND IN BINÄREN GEMISCHEN VON EINATOMIGEN GASEN

Zusammenfassung—Die Wärmeleitung in einfachen einatomigen Gasen und in binären Gemischen einatomiger Gase wurde experimentell und theoretisch untersucht für Temperaturdifferenzen bis zu 300 K und für einen Knudsen zahlbereich von $10^{-4} < Kn < 10^3$. Die Experimente wurden in einer Wärmeleitzelle mit horizontalen parallelen ebenen Platten durchgeführt, und die theoretischen Ergebnisse wurden mittels vier bzw, acht Momentengleichungen als Lösungen der Boltzmanngleichung erhalten.

ТЕПЛООБМЕН В ПРОСТЫХ ОДНОАТОМНЫХ ГАЗАХ И ИХ БИНАРНЫХ СМЕСЯХ

Аннотация — Проведено экспериментальное и теоретическое исследование кондуктивного теплообмена в простых одноатомных газах и в их смесях при разностях температур до 300 К в диапазоне изменения числа Кнудсена $10^{-4} < \mathrm{Kn} < 10^3$. Эксперименты проводились в ячейке теплопроводности, выполненной по методу плоского горизонтального слоя. Приводятся теоретические расчеты для решений уравнения Больцмана в приложении четырех и восьми моментов.